1. The curve C has the equation $ye^{-2x} = 2x + y^2$.

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The point P on C has coordinates (0, 1).

(b) Find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)
(Total 9 marks)

2. $$f(x) = (x^2 + 1) \ln x, \qquad x > 0.$$

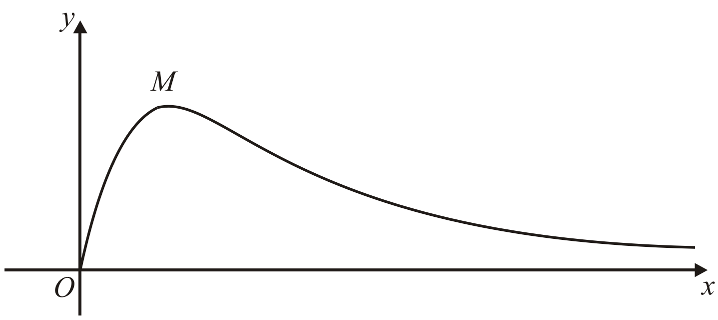
(a) Use differentiation to find the value of $f'(x)$ at $x = e$, leaving your answer in terms of e.

(4)

(b) Find the exact value of $\int_1^e f(x)\,dx$

(5)
(Total 9 marks)

3.



The diagram above shows the curve with equation $y = x^{\frac{1}{2}} e^{-2x}$.

(a) Find the x-coordinate of M, the maximum point of the curve.

(5)

The finite region enclosed by the curve, the x-axis and the line $x = 1$ is rotated through 2π about the x-axis.

(b) Find, in terms of π and e, the volume of the solid generated.

(7)
(Total 12 marks)

1. (a) $e^{-2x}\dfrac{dy}{dx} - 2ye^{-2x} = 2 + 2y\dfrac{dy}{dx}$ A1 correct RHS *M1 A1

$\dfrac{d}{dx}(ye^{-2x}) = e^{-2x}\dfrac{d}{dx} - 2ye^{-2x}$ B1

$(e^{-2x} - 2y)\dfrac{dy}{dx} = 2 + 2ye^{-2x}$ *M1

$\dfrac{dy}{dx} = \dfrac{2+2ye^{-2x}}{e^{-2x} - 2y}$ A1 5

(b) At P, $\dfrac{dy}{dx} = \dfrac{2+2e^{o}}{e^{o}-2} = -4$ M1

Using $mm' = -1$

$m' = \dfrac{1}{4}$ M1

$y - 1 = \dfrac{1}{4}(x - 0)$ M1

$x - 4y + 4 = 0$ or any integer multiple A1 4

Alternative for (a) differentiating implicitly with respect to y.

$e^{-2x} - 2ye^{-2x}\dfrac{dx}{dy} = 2\dfrac{dx}{dy} + 2y$ A1 correct RHS *M1 A1

$\dfrac{d}{dy}(ye^{-2x}) = e^{-2x} - 2ye^{-2x}\dfrac{dx}{dy}$ B1

$(2 + 2ye^{-2x})\dfrac{dx}{dy} = e^{-2x} - 2y$ *M1

$\dfrac{dx}{dy} = \dfrac{e^{-2x} - 2y}{2+2ye^{-2x}}$

$\dfrac{dy}{dx} = \dfrac{2+2ye^{-2x}}{e^{-2x} - 2y}$ A1 5

[9]

2. (a) $f'(x) = (x^2 + 1) \times \dfrac{1}{x} + \ln x \times 2x$ M1 A1

$f'(e) = (e + 1) \times \dfrac{1}{e} + 2e = 3e + \dfrac{1}{e}$ M1 A1 4

(b) $(\frac{x^3}{3}+x)\ln x - \int(\frac{x^3}{3}+x)\frac{1}{x}dx$ M1 A1

$= (\frac{x^3}{3}+x)\ln x - \int(\frac{x^3}{3}+1)dx$

$= \left[(\frac{x^3}{3}+x)\ln x - (\frac{x^3}{9}+x)\right]_1^e$ A1

$= \frac{2}{9}e^3 + \frac{10}{9}$ M1 A1 5

[9]

3. (a) $\frac{dy}{dx} = 2e^{-2x}\sqrt{x} + \frac{e^{-2x}}{2\sqrt{x}}$ M1 A1 A1

Putting $\frac{dy}{dx}$ = 0 and attempting to solve dM1

$x = \frac{1}{4}$ A1 5

(b) Volume = $\pi\int_0^1(\sqrt{x}e^{-2x})^2\,dx = \pi\int_0^1 xe^{-4x}dx$ M1 A1

$\int xe^{-4x}dx = -\frac{1}{4}xe^{-4x} + \int\frac{1}{4}e^{-4x}dx$ M1 A1

$= -\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x}$ A1 ft

Volume = $\pi\left[-\frac{1}{4}e^{-4} - \frac{1}{16}e^{-4}\right] - \left[-\frac{1}{16}\right] = \frac{\pi}{16}[1-5e^{-4}]$ M1 A1 7

[12]

1. As noted above work on this topic has shown a marked improvement and the median mark scored by candidates on this question was 8 out of 9. The only errors frequently seen were in differentiating $y\,e^{-2x}$ implicitly with respect to x. A few candidates failed to read the question correctly and found the equation of the tangent instead of the normal or failed to give their answer to part (b) in the form requested.

2. The product rule was well understood and many candidates correctly differentiated f(x) in part (a). However, a significant number lost marks by failing to use ln e = 1 and fully simplify their answer.

Although candidates knew that integration by parts was required for part (b), the method was not well understood with common wrong answers involving candidates mistakenly suggesting that $\int \ln x \, \mathrm{d}x = \frac{1}{x}$ and attempting to use $u = x^2 + 1$ and $\frac{\mathrm{d}v}{\mathrm{d}x} = \ln x$ in the formula $\int u \frac{\mathrm{d}v}{\mathrm{d}x} \mathrm{d}x = uv - \int v \frac{\mathrm{d}u}{\mathrm{d}x} \mathrm{d}x$.

Candidates who correctly gave the intermediate result $\left[\left(\frac{x^3}{3} + x\right)\ln x\right]_1^{\mathrm{e}} - \int_1^{\mathrm{e}} \left(\frac{x^3}{3} + x\right)\frac{1}{x}\,\mathrm{d}x$ often failed to use a bracket for the second part of the expression when they integrated and went on to make a sign error by giving $-\frac{x^3}{9} + x$ rather than $-\frac{x^3}{9} - x$.

3. This question involved differentiation using the product rule in part (a) and integration using parts in part (b). It was answered well, with most of the difficulties being caused by the use of indices and the associated algebra. Some candidates wasted time in part (a) by finding the y-coordinate which was not requested. A sizeable proportion of the candidates misquoted the formula for volume of revolution.